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PARAMETRIC DESIGN AND OPTIMIZATION OF A STANDARD SPUR GEAR WITH CIRCULAR FILLET USING FINITE ELEMENT METHOD

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ABSTRACT

Gears are toothed cylindrical wheels used for transmittal mechanical power from one rotating shaft to a different and is one of the foremost important parts in a mechanical power gear. Due to their high degree of compactness and reliability it's expected that gears can predominate because the best suggests that of power transmission in close to future. Increase in demand for accurate and quiet power transmission in machines, generators and vehicles has resulted in the need for a lot of precise analysis of drugs characteristics. In auto business as lighter vehicles continue to be in demand, highly reliable and light-weight weight gears became necessary. Designing extremely loaded spur gears for power transmission systems that are each robust and quiet, requires analysis ways that will simply be enforced and conjointly give info on contact and bending stresses, along with transmission errors. Transmission error results due to two main factors i.e. manufacturing quality or mounting errors and elastic deflection underneath load. Hence, compensation for transmission errors by modifying gear teeth needs to be thought-about by gear designers. Transmission error is considered to be one the most contributors to noise and vibration in a gear set. If a pinion and gear have ideal involute profiles running with no loading torque they must in theory run with zero transmission error.

Keywords: *FEA; static analysis; spur gear; bending stress; ANSYS; optimization.*

I. INTRODUCTION

Gears are toothed wheels used to transmit motion and power between rotating shafts by means of engagement of its teeth. Due to its exact ratio of transmission gears are indispensable parts in precision tools, equipment and machinery. The increased demand for efficient power transmission in all the sectors including automotive, machineries, generators etc. has resulted into more focus on the precise study of the various characteristics of gears and its attributes. The demand for fuel and energy efficient devices calls for lighter and stronger gears. Gears that are used in aviation must have light structure for reduced weight. Thin rimmed gears are therefore widely used.Lately, this kind of gear has been finding wider applications in cars and other general machines. [21]

Due to the thin cross section of the gear as a result of material removal the weight of the gear reduces but the bending strength of the gear reduces. This trade-off between mass and strength and its relation with rim thickness is studied in the current work. Analytical methods can be employed to determine optimum values for rim thickness, mass and bending strength. This can provide accurate results. Since number of iterations involved are huge analytical method for optimization could be time consuming finite element analysis is preferred for optimization problems. With the advent of more powerful computers and their ability to process more instructions per second one can expect more accurate and faster results given that the assumptions and boundary conditions are correct and the variables are practical. [1-2]

In this work, static structural analysis is carried out on a single spur gear and its dimensions are optimized to reduce its bending stress. Since gear drives operate with a power efficiency significantly higher than any other mechanical drive, or any electrical, hydraulic or pneumatic power transmission, they have thewidest use in transforming rotary motion from the prime mover to the actuator, and their importance is growing day by day. Although efficiency is not the only criterion for choosingthe type of transmission, the gear drive, due to its robustness and operational reliability, presents an inevitable component of most mechanical engineering systems. Gear drives are known to be highly demanding in design, manufacture, control and maintenance. [4]

Many organizations, including the American Gear Manufacturer's Association (AGMA) have sought to standardize the gear design process by developing their own formulas for gear design. The major changes over Lewis' original equations are the ability to take into account the geometrical complexity of the gear tooth, as well as the actual location of the contact. With the advent of Finite Element Analysis and Computer Aided Design (FEA and CAD) the ability to quickly and accurately design gears has been greatly improved upon. With modern CAD programs a typical gear and pinion can be modelled relatively easy. With better computing power, FEA software can quickly and efficiently analyze the stresses and contact pressures in gear pairs. These tools make the design and analysis stage much cheaper and faster for the design engineer. Because actual experiments are costly and take large amounts of time, a repeatable and accurate design tool is crucial for real world application. As the distance between the gear and pinion's axis of rotation increases the bending stress and contact pressure rise. A relatively small rise in bending stress can cause a gear tooth to fail at lower cycle numbers than the design calls for. With an increase in the contact pressure between a gear and pinion wear will be increased as well. With increased wear, gear failure may occur sooner by pitting, corrosion, and adhesive wear. Although much study has been done on the mechanisms of gear wear and the major contributors to that wear, there is still a lack of understanding when it comes to the tolerances and how they affect the wear characteristics.

II. METHOD OF ANALYSIS AND ASSUMPTIONS

A single spur gear is used with the load applied at tooth tip. The standard dimensions for a 20 pressure angle and module 1 are considered. A load of 200 N is applied at the tool tip.

Table 1 Structure of a spur gear

Standard involute spur gears	
Module m	1
Pressure angle φ	20°
Number of teeth N	20
Face width b	8 mm
Profile shift coefficient	0
Shaft diameter	7 mm
Dedendum	1.25 mm
Addendum	1 mm
Root radius	0.38 mm
Tip diameter	22 mm
Root diameter	17.5 mm
Tooth thickness	1.5708 mm
Rim diameter	16.5 mm

Table 2 Mechanical properties of gear

Material	Structural Steel
Volume	1250.3 mm^3
Mass	9.815×10^{-3} kg
Young's modulus	2×10^5 MPa
Poisson's Ratio	0.3

While performing the analysis few assumptions were made [10]:

1. The full load is applied to the tip of a single tooth in static condition.
2. The radial component is negligible.
3. The load is distributed uniformly across the full face width.

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4. Forces due to tooth sliding friction are negligible.
5. Stress concentration in the tooth fillet is negligible.

III. PROCEDURE

The objective of this analysis is to find optimum value of the rim diameter so that the gear does not fail for the applied bending stress. The causes of gear failure are pitting through contact or fracture through bending stress.

Parametric design

To start generating the curve from A

$x = r\cos t + rt\sin t = r(cos\ t + t\ sin\ t)$---------------------- (4.1)

$y = r\ sin\ t - rt\ cos\ t = r(sin\ t - t\ cos\ t)$--------------------- (4.2)

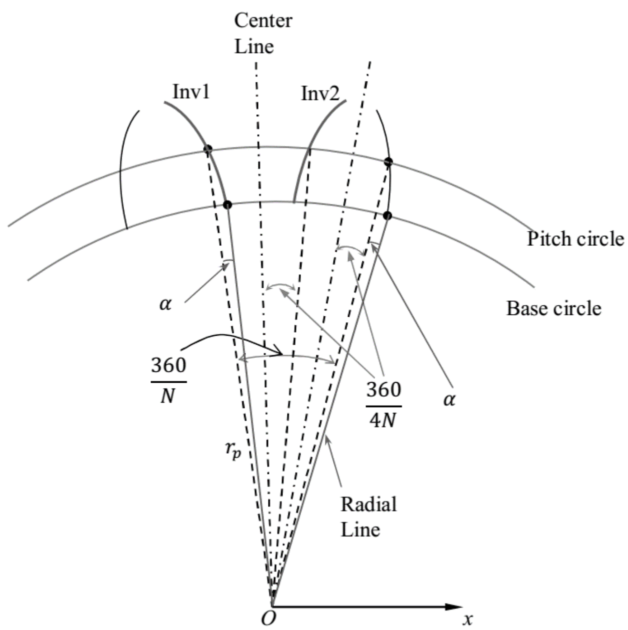

Fig. 1 Spur gear tooth profile generation

For the curve starting from C

$x = r\left(cos\ \left(t + \frac{\pi}{2}\right) + t\ sin\ \left(t + \frac{\pi}{2}\right)\right)$---------------------------- (4.3)

$y = r\left(sin\ \left(t + \frac{\pi}{2}\right) - t\ cos\ \left(t + \frac{\pi}{2}\right)\right)$---------------------------- (4.4)

Where t is the angle parameter in radians

If we would like to start the involute from point Cand direction of the involute as shown, then the parametric equation is

$x = r\big(cos(-t - \beta) - t\ sin(-t - \beta)\big)$------------------ (4.4)

$x = r\big(sin(-t - \beta) + t\ cos(-t - \beta)\big)$------------------ (4.5)

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Impact Factor- 5.070

Unit for β needs to be in radian when entering the expression in Solidworks. We can use the base circle diameter dimension in the expression to drive the involute. This way, the involute will be updated once the base circle diameter is changed.

The expressions for creating the two involute curves as inserted in the SolidWorks are

For first involute profile

X(t)=”D2@Sketch2”*0.5*(cos(t)+t*sin(t))-------------------(4.6)
Y(t)=”D2@Sketch2”*0.5*(sin(t)-t*cos(t))--------------------(4.7)

For second involute profile

X(t)=”D2@Sketch2”*0.5*(cos(-t-2*”D4@Sketch2”*pi/180)-t*sin(-t-2*”D4@Sketch2”*pi/180))-----------------------------------(4.8)

Y(t)=”D2@Sketch2”*0.5*(sin(-t-2*”D4@Sketch2”*pi/180)+t*cos(-t-2*”D4@Sketch2”*pi/180))------------------------------------(4.9)

Where “D2@Sketch2” is the dimension for base circle diameter. “D4@Sketch2” is for $\frac{360}{4N} - \alpha$.

Problem Formulation

A solid 3D model of a spur gear is constructed in Solid works. It is then exported to ANSYS and required boundary conditions are applied. The results are generated using solver for maximum bending stress. The variables were then optimized for maximum rim diameter, minimum bending stress and minimum mass.

Initially a force of 200 N is applied at the tip of a tooth. The bending stresses can be validated by the use of AGMA bending stress relation [10]

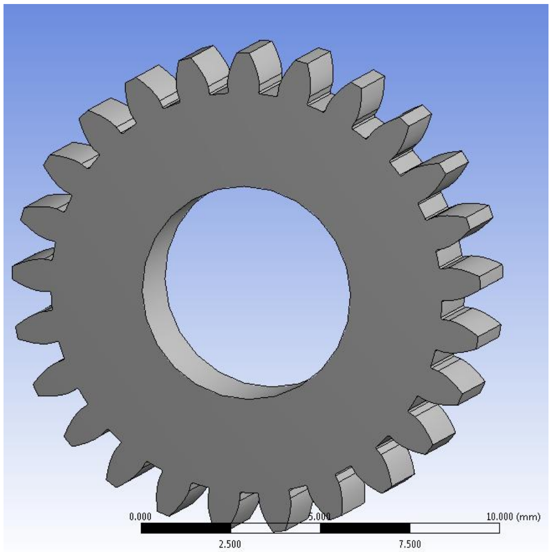

Fig. 2 Spur gear with fillet radii and applied load

ISSN 2348 – 8034
Impact Factor- 5.070

$$\sigma_b = \frac{F_t}{b \times m \times J} K_v \times K_O \times K_m$$

Where,

σ_b=*Bending stress*
F_t=*Tangential force=200 N*
b=Face width=8 mm
m=Module= 1
J=Geometry factor=0.24
K_v=*Velocity factor=1*
K_o=*Overload factor=1.3*
K_m=*Load distribution factor=1.3*

$$\sigma_b = \frac{200 \times 1 \times 1.3 \times 1.3}{8 \times 1 \times 0.24} = 176.0416\ MPa$$

The analytical value of bending stress calculated using AGMA formula is in agreement with the Finite Element Method value.

Table 3 Bending stresses values comparison

	Bending stress	Value, MPa	Percent error
1	AGMA	176.0416	
2	ANSYS	176.57	
3			0.29

Boundry Conditions

In ANSYS Design modeller workbench, the boundary conditions of load and support are applied as shown in figure below. The mesh of type   is used. The problem is solved for finding maximum and minimum bending stress.

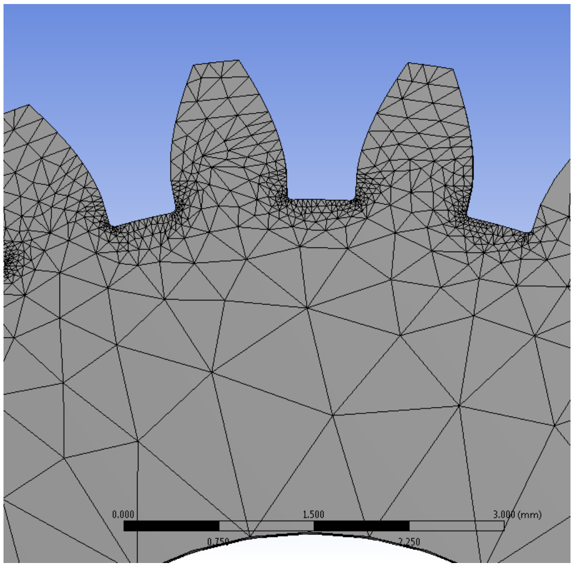

Fig. 3 Spur gear with applied load and boundary conditions

ISSN 2348 – 8034
Impact Factor- 5.070

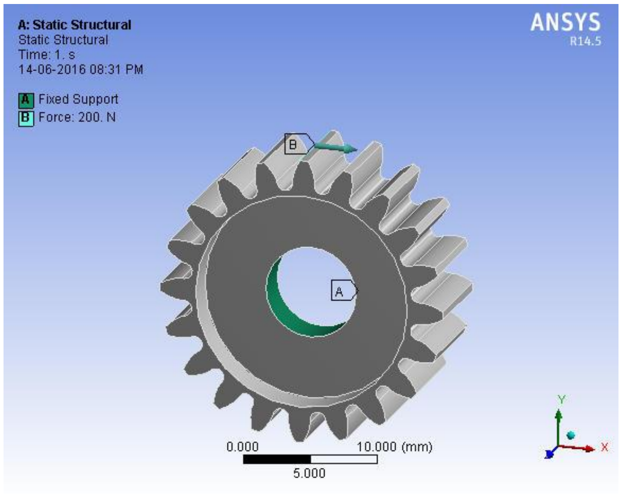

Fig. 4 Spur gear with applied load and boundary conditions

IV. RESULT AND DISCUSSION

After finding out the value of maximum and minimum bending stresses, the maximum value of bending stress, and mass of gear and rim diameter are considered as variables. Out of which the rim diameter is the input variable and the remaining two output variables. The level of interdependencies of these variables is characterized in the following matrix.

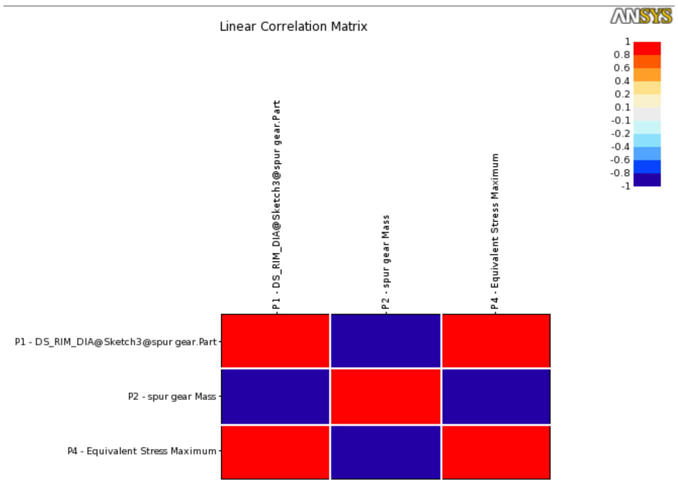

Fig. 5 Correlation Matrix for various parameters involved

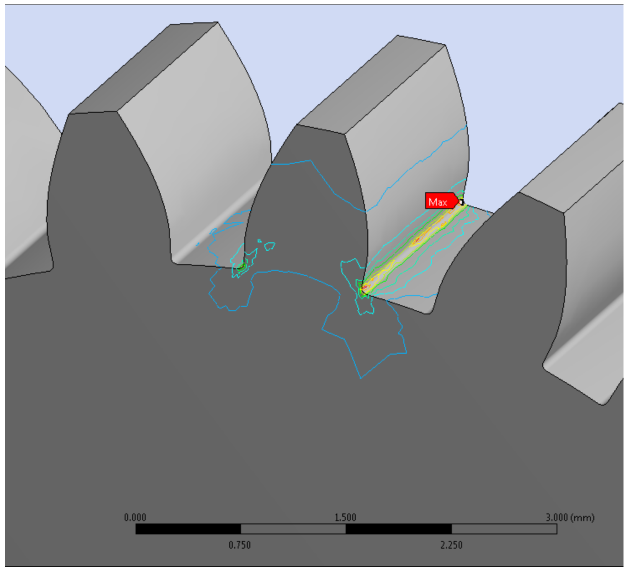

Fig. 6 Bending stress distribution

At least 100 random values of the input variable are considered for optimization in the range of 17-14.5 mm and top 30 points were used for optimization as shown in table 3.

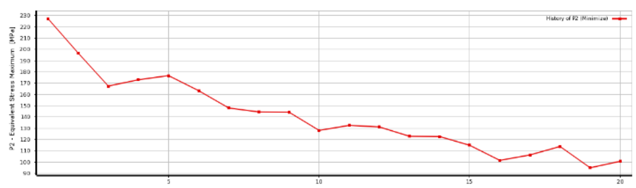

Fig.7 Bending stress vs Fillet radii

Table 4 Design Points

Table of Schematic B2: Optimization

	A	B	C
1	Name	P1 - FBlend1.FD1	P2 - Equivalent Stress Maximum (MPa)
2	1	0.05625	227.31
3	2	0.06875	196.83
4	3	0.08125	167.52
5	4	0.09375	173.11
6	5	0.10625	176.81
7	6	0.11875	163.42
8	7	0.13125	148.03
9	8	0.14375	144.53
10	9	0.15625	144.35
11	10	0.16875	128.14
12	11	0.18125	132.62
13	12	0.19375	131.22
14	13	0.20625	122.94
15	14	0.21875	122.73
16	15	0.23125	114.92
17	16	0.24375	101.53
18	17	0.25625	106.35
19	18	0.26875	113.87
20	19	0.28125	94.958
21	20	0.29375	100.72

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Corresponding response of the output variables was noted and candidate points were shortlisted. The points of convergence graph indicates the approximate design point at which the optimum value is going to lie.

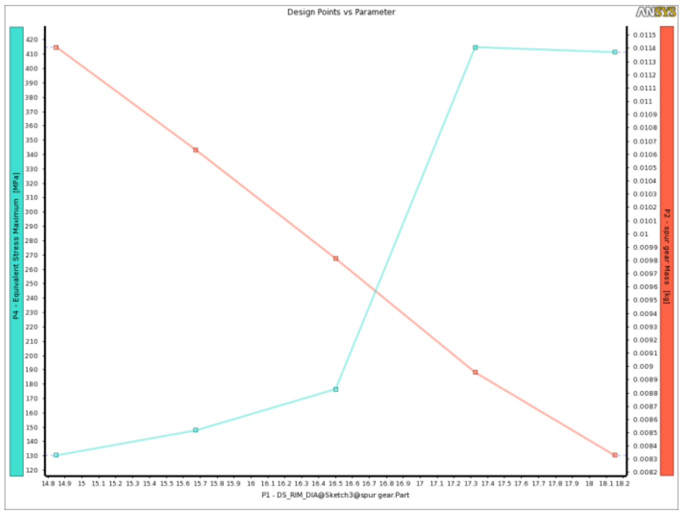

Fig. 7 Design points vs Parameters

Collective results after optimization in comparison with the original result is tabulated below.

Table 5 Optimized candidate points

Table of Schematic B2: Optimization , Candidate Points

	A	B	C	D	E
1	Reference	Name	P1 - FBlend1.FD1	P2 - Equivalent Stress Maximum (MPa)	
2				Parameter Value	Variation from Reference
3	◉	Candidate Point 1	0.28125	★★★ 94.958	0.00 %
4	○	Candidate Point 2	0.24375	★★★ 101.53	6.93 %
5	○	Candidate Point 3	0.21875	★★ 122.73	29.25 %
*		New Custom Candidate Point	0.1725		

V. CONCLUSION

In this work, it is shown that multi variable optimization can be used to optimize any number of input and output variables quickly. With reference to table no. 4 it is quite evident that there is 29.25 % reduction in the bending stress with very small rise in the mass and rim diameter. This shows that the dimensions considered were optimized. Though the determining factors for accuracy includes the upper and lower limits of the variables, the method of optimization used and practically choosing the conditions of optimization. The determining factors for the accuracy of the results are the finesse of mesh, point of application of load: there is slight variation in the output bending stress if it is applied at HPSTC or tooth tip and whether the analysis is static structural or explicit dynamic. The change in root radius influences the bending stress of the gear. An optimum value of the root radius can be calculated for the given gear dimensions.

Further this result can be verified and compared with various methods of optimization present in the software for better efficiency and can also be optimized for agreement with six sigma standards.

REFERENCES

1. *Li, Shuting, 2002,"Gear Contact Model and LoadedTooth Contact Analysis of aThree-Dimensional,Thin-Rimmed Gear," Journal of Mechanical Design, SEPTEMBER 2002, Vol. 124/511*
2. *Li, Shuting, 1998, ''Fundamental Studies of Analyzing the Tooth-Load Distribution of Three-dimensional, Thin-Rimmed Gears with Assembly Errors,'' Doctoral Dissertation, Yamaguchi University.*
3. *"Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth." ANSI/AGMA 2001- B88, American Gear Manufacturers Association, Alexandria, VA, 1990.*
4. *Damir Jelaska, "Gears and Gear Drives," University of Split, Croatia, 2012 John Wiley & Sons Ltd. ISBN 978-1-119-94130-9*
5. *Ji, Mingang, 1980, ''A Method for Contact Analysis of Elastic Bodies using FEM and A Mathematical Programming Method,'' Research Report of Northwestern Polytechnical University, No. SHJ8032, pp. 1–18.*
6. *Prabhu, M. S., and Houser, D. D. R., 1996, ''A Hybrid Finite Element Approach for Analyzing the Load Distribution and Transmission Error in Thin Rimmed Gears,'' VDI-Ber., 1230, pp. 201–210.*
7. *Wilcox, L., and Coleman, W., 1973, "Application of Finite Elements to the analysis of gear tooth stresses", ASME Journal of Engineering for*
Industry, vol. 95, pp. 1139-1148.
8. *Ishida, T., Hidaka, T., and Takizawa, H., 1985, ''Bending Stress Analysis ofIdle Gear with Thin Rim Effects of Rim Thickness on Stresses,'' Bull. JSME, 51(462), pp. 359–365.*
9. *Kawalec, A., Wiktor, J., & Ceglarek, D. 2006. "Comparative Analysis ofTooth- Root Strength Using ISO and AGMA Standards in Spur and*
Helical Gears With FEM-based Verification" Journal of MechanicalDesign, 128(5), 1141.
10. *Andrew Wright, 2013, "A Comparison of the Tooth-Root Stress and Contact Stress of an Involute Spur Gear Mesh as Calculated by FEM*
and AGMA Standards", Master of Engineering paper, Rensselaer Polytechnic Institute Hartford, CT
11. *http://nptel.iitm.ac.in/courses/IITMADRAS/Machine_Design_II/pdf/2_7.pdf*
12. *Lewicki, D., G., Ballarini, R., "Rim Thickness Effects on Gear Crack Propagation Life", International Journal of Fracture, Vol. 87, 1997, pp 59-86.*
13. *Wawrzynek, P. A. "Discrete Modeling of Crack Propagation: Theoretical. Aspects and Implementation Issues in.Two and Three Dimensions." Ph.D. Dissertation, Cornell University, 199L*
14. *Flašker, J., Pehan, S.,"Crack Propagation in Tooth Root with Variable Loading", Communications in Numerical Methods in Engineering, Vol. 9, 1993, pp 103-110.*
15. *He, Qiong, 1981, ''An Improved Three-Dimensional, 8-nodes NonCompatibility Solid Element (In Chinese),'' Journal of Shanghai Jiaotong University, 4.*
16. *Kramberger, J., Potrc, I., Flašker, J., "Crack Propagation Analysis of thin Rim Gears using Boundary Element Method", Proceedings DESIGN 2000. Cavtat-Dubrovnik Croatia, 2000, pp 739-744.*
17. *Ishida, T., and Hidaka, T., 1994, ''Stress of Planet Gear with Thin Rim,'' GearTechnology, 11(2), pp. 26–31.*
18. *Cavdar, K., Karpat, F., & Babalik, F. C. 2005. "Computer Aided Analysis of Bending Strength of Involute Spur Gears with Asymmetric*
Profile." Journal of Mechanical Design, 127(3), 477.
19. *Inoue, K. et al. "Fracture Mechanics Based Evaluation of Strength. of Carburized Gear Teeth." Proceedings of the JSME International Conference on Motion and Power Transmissions. Hiroshima, Japan, Nov" 1991, pp. 80*
20. *AGMA standard, 2004, "Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth" ANSI/AGMA 2001- -D04 [Revision of ANSI/AGMA 2001--C95], American Gear Manufacturers Association, 500 Montgomery Street, Suite 350,*
Alexandria, Virginia 22314, ISBN: 1--55589--839—4
21. *Rathore, R. and Tiwari, A., "Bending Stress Analysis & Optimization of Spur Gear," IJERT, ISSN: 2278-0181, Vol. 3 Issue 5, May - 2014*
22. *Norton, R.L., 1996, "Machine Design: An Integrated Approach", New Jersey: prentice- Hall Inc. ISBN-10: 0135554756*

[NC-Rase 18] ISSN 2348 – 8034
DOI: 10.5281/zenodo.1494024 Impact Factor- 5.070

23. *Guingand, M., de Vaujany, J. P., and Icard, Y., "Analysis and Optimization of the Loaded Meshing of Face Gears", Journal of Mechanical Design, vol. 127, pp. 135-143, 2005.*
24. *Budgell, Peter, 2003, "Finite Element Analysis and Optimization Introduction", ROI Engineering Inc, Ontario, Canada.*
25. *Zeiping Wei', 2004, "Stress and Deformations in involute Spur gears by Finite Element Method", M.Sc. Paper, University of Saskatchewan.*
26. *Daniewicz, S, R. et al, "The Stress Intensity Factor and Stiffness for a Cracked Spur Gear Tooth," Journal of Mechanical Design, Vol. 116, No.3, Sept., 1994, 9. Flasker, 1. and S. Pehan, "Crack Propagation in Tooth Root with Variable Loading .." Communications in Numerical Methods in Engineering, Vol.. 9, No.2, Feb., 1993,. pp. 103-110.*
27. *Deb, K., 2001, "Multiobjective Optimization Using Evolutionary Algorithms", Wiley, New York. ISBN: 978-0-471-87339-6*
28. *AGMA standard Information Sheet "Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical and Herringbone Gear Teeth Gear Teeth" AGMA 908---B89, April 1989 (Revision of AGMA 226.01 1984), American Gear ManufacturersAssociation, 500 Montgomery Street, Suite 350, Alexandria, Virginia 22314, ISBN: 1---55589---525---5.*